An Analytical Expression for Friction Factor

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It is convenient to have a single equation from which friction factor may be calculated. This equation, to be fully useful, should accommodate the full range of Reynolds numbers and relative roughnesses which are of interest. Such an equation is particularly convenient for use with computer programs, where the friction factor must be calculated as a subroutine from other information derived by the program.

An equation has been available for some time for the completely smooth tube based on the integration of the dimensionless velocity distribution equations of Von Karmon (*1*). Similarly, using his experimental data, Nikuradse developed equations for the totally rough, fully turbulent flow regime (*2*). Moody (*3*) has combined these results with Colebrook's (*4*) transition function, and with experimental data on commercial roughness, to prepare a chart relating friction factor and Reynolds number. The curves have been positioned to give best agreement with experimental data, each curve representing one relative roughness. This graph has been reproduced in a number of places (*5, 6*) and is now well accepted.

Recent work (*7*) on boundary-layer flow over rough surfaces has presented new understanding of roughness and suggests a method for recorrelation of the Moody type of plot in a simpler way. The reader is referred to Clauser's article for details of the development. In short the effect of wall roughness is expressed as a shift in the velocity distribution curve, when this curve is expressed in the usual dimensionless coordinates. For a smooth tube the velocity distribution is given as

$$\left(\frac{u}{u_*}\right)_{\text{smooth}} = A \ln \frac{u_* \rho y}{u} + c \quad (1)$$

For a rough tube this becomes

$$\left(\frac{u}{u_*}\right)_{\text{rough}} = A \ln \frac{u_* \rho y}{u} + c - \frac{\Delta u}{u_*} \quad (2)$$

where $\Delta u/u_*$ is the shift due to roughness. Furthermore Clauser (*7*) demonstrates that this velocity shift depends on the roughness Reynolds number $u_* \rho k/u$.

If Equation (1) is used to set up the differential volumetric flow and this is integrated over the tube crosssectional area, then resistance equation for a smooth tube is obtained:

$$\left(\frac{1}{\sqrt{f}}\right)_{\text{smooth}} = P \ln (N_{Re}\sqrt{f}) + Q \quad (3)$$

where P and Q are combined constants from the velocity distribution Equation (1) and from integration. If Equation (2) is used in a similar manner, the following is obtained.

$$\left(\frac{1}{\sqrt{f}}\right)_{\text{rough}} = P \ln (N_{Re}\sqrt{f}) + Q - \frac{\Delta u}{u_*} \quad (4)$$

It is now necessary only to characterize $\Delta u/u_*$ to obtain a final equation for the friction factor in rough tubes from Equation (4).

It has been shown (*7*) that $\Delta u/u_*$ depends only on $u_* \rho k/u$. Note that this roughness Reynolds number can be modified as follows from the definition of friction velocity and Reynolds number.

$$\frac{u_* \rho k}{u} = N_{Re} \frac{k}{D} \sqrt{f/2} \quad (5)$$

Values of $\Delta u/u_*$ were calculated for various Reynolds numbers, pipe sizes, and relative roughness values by means of the data represented by the Moody chart and Equation (4). All quantities were obtained from the chart and the shift then calculated. Each value of $\Delta u/u_*$ was plotted against the corresponding roughness Reynolds number of Equation (5). The result is shown in Figure 1. Data taken from all regions of the Moody chart are characterized by a single curve. It now remains only to fit this curve empirically and substitute into Equation (4) to obtain an analytical expression for the friction factor at any roughness condition and Reynolds number. This result with the numerical value of all constants substituted is

$$\frac{1}{\sqrt{f/2}} = 1.03 + 5.76 \log \left(\frac{N_{Re}}{2}\sqrt{f/2}\right) - 1.75 \log R - 1.10 (\log R)^2 \quad (6)$$

where $R = N_{Re} \frac{k}{D} \sqrt{f/2}$. When P is less than 0.05, the smooth tube equation should be used:

$$\frac{1}{\sqrt{f/2}} = 1.75 + 5.76 \log \left(\frac{N_{Re}}{2}\sqrt{f/2}\right) \quad (7)$$

Agreement of (6) and (7) with the prediction of the Moody chart is, at all conditions, better than 2%.

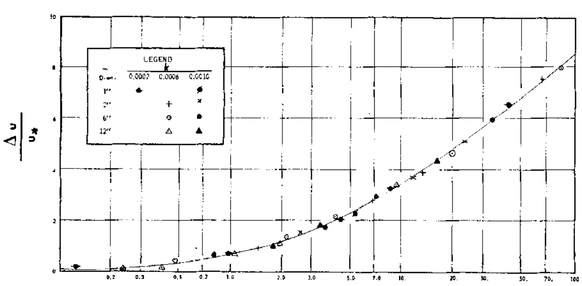

Fig. 1. Velocity shift correlation.

NOTATION

A, C, E, P, Q	= experimental constants
D	= tube diameter
f	= friction factor
k	= absolute roughness magnitude
N_{Re}	= Reynolds number
R	= roughness Reynolds number
u	= local velocity
u_*	= friction velocity
y	= distance from the solid boundary
ρ	= density
μ	= viscosity
τ_o	= wall shear
$\Delta u/u_*$	= shift due to roughness

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